APPLICATION OF SEMI-MARKOV PROCESSES AND UNIVERSAL GENERATING FUNCTION TECHNIQUE TO MULTI-STATE SYSTEM RELIABILITY EVALUATION

A. Lisnianski, *The Israel Electric Corporation Ltd, Haifa, Israel*
e-mail: anatoly-l@iec.co.il, lisn52@hotmail.com

Abstract

Key words and phrases: Multi-state system; Performance; Universal generating function; Reliability; Semi-Markov process.

Traditional binary reliability models allow only two functional states for a system and for each of its components: perfect functionality (UP) and complete failure (DOWN). A significant research effort has been made until now to extend the reliability assessment from binary-state system to multi-state system (MSS), where the system and its components have a range of performance levels – from perfect functionality to complete failure. Reliability models were extended from binary state (UP or DOWN) to finite number of performance levels (Multi-State System models).

A semi-Markov processes method is a powerful approach and its application to reliability evaluation gives the opportunity to get analytical solution for a MSS where failure and repair times may be arbitrary (non-exponentially) distributed. The main difficulty of the semi-Markov processes application to reliability evaluation for complex MSS is the "dimension damnation". Indeed, the number of integral equations in the system that should be solved using semi-Markov approach is equal to the total number of MSS states. For MSS consisting of n different repairable components where every component j has k_j different performance levels one will have a system of $\prod_{j=1}^{n} k_j$ integral equations in order to find states probabilities for the MSS by using straightforward semi-Markov method. This number can be very large even for relatively small MSS. Although the modern software tools can provide solutions for systems of integral equations, building the state-space diagram and deriving the corresponding system of integral equations is a difficult non-formalized process that may cause numerous mistakes.

An application of the Universal Generating Function (UGF) technique for reducing the dimension of system of integral equations is considered in this paper. The proposed method is

named as *combined UGF and semi-Markov process technique*. The technique can be used for MSS consisting of statistically independent repairable elements.

At the first stage smaller semi-Markov models should be built for each component of the MSS. For component j the semi-Markov model consists of k_j integral equations. By solving these equations, the performance probability distribution can be obtained for every component at each time instant t. Based on these probability distributions, UGF for each component can be defined.

At the second stage, based on composition operators over UGF of individual components and their combinations in the entire MSS structure, one can obtain the resulting UGF for the entire MSS by using simple algebraic operations. This UGF defines the output performance distribution for the entire MSS at each time instant t. MSS reliability measures such as instantaneous mean output performance, instantaneous performance deficiency, instantaneous availability etc. can be easily derived from this output performance distribution.

The advantages of the proposed approach are:

1. Simplification of semi-Markov model building. Instead of the building of complex semi-Markov model for the entire MSS, one should built n separate relatively simple semi-Markov models for system components.
2. Simplification of the process of solving the system of equations. Instead of solving one high-order system of $\prod_{j=1}^{n} k_j$ integral equations one has to solve n low-order systems with total number of equations $\sum_{j=1}^{n} k_j$. In each system the number of equations is lower or equal than $\max_j\{k_j\}$.

Numerical example is also presented in the paper in order to illustrate the proposed approach.